Synchrotron Light in a Nutshell

Giorgio Margaritondo
Institut de Physique Appliquée, Ecole Polytechnique Fédérale de Lausanne,
CH-1015 Lausanne, Switzerland

Abstract
I present an extremely simple derivation of the underlying physics and basic equations of the five most important properties of synchrotron light sources.

The use of synchrotron sources is widespread in different disciplines, and in a state of explosive growth. Paradoxically, many users do not understand the simple physical causes of their amazing properties. In a recent article, I demonstrated[1] that such properties can be derived with a very simple approach — and no integrals at all..

I now present an even simpler derivation for a subset of the synchrotron light properties. This allows the underlying physics to stand up clearly, not cluttered by mathematical formalism.

The treated propertied are (1) the spectrum (peak and bandwidth) and the angular spread of an undulator; (2) the spectrum (peak and bandwidth) and the angular spread of a bending magnet and of a wiggler; (3) flux and brightness; (4) polarization (5) coherence. I will assume that the reader is already qualitatively and generally familiar with the components of a synchrotron source, i.e., the storage ring with its bending magnets and insertion devices (wigglers and undulators).

Before going into the detailed discussion, we need a minimum of background. The most important physical point underlying the discussion is that *synchrotron light production is always achieved by exploiting the combination of two relativistic effects, for example Lorentz contraction and the Doppler shift*. Thus, we must recall the simple relativistic rules for changing the reference frame.

As shown in Fig. 1, in the laboratory frame we call x′ the coordinate along the electron beam motion, y′ the perpendicular coordinate in the plane of the storage ring and θ′ the light emission angle with respect to the x-axis. The corresponding coordinates in the source frame (electron frame) are: x, y and θ.

1. G. Margaritondo, J. Synchrotron Radiation **2**, 148 (1995).

Note that the "electron frame" is not the frame moving with the electron (where the electron would have no acceleration and therefore would emit no light). It is instead the inertial frame moving along the x-axis with the same instantaneous speed u as the light-emitting electron.

In our discussion we will use the standard relativistic factors $\beta = u/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. One should keep in mind that $\gamma = mc^2/m_oc^2$, and therefore is the energy of the light-emitting electron (the accelerator energy) expressed in units of m_oc^2, the electron rest energy. In the appendix, we report a few simple mathematical tricks that are used in the following discussion.

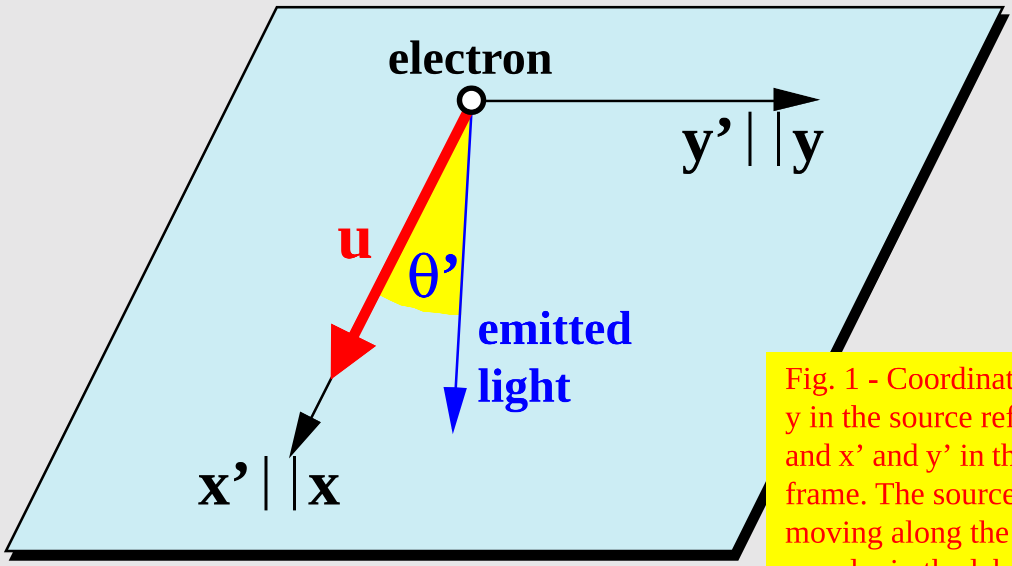


Fig. 1 - Coordinate systems: x and y in the source reference frame, and x' and y' in the laboratory frame. The source (electron) is moving along the x-axis, at the speed u in the laboratory frame.

Undulators

Underlying physics

- The peak emission wavelength of an undulator is given by the undulator period, shortened first by the Lorentz contraction and then by the Doppler shift.
- The corresponding shortening factors must take into account the angular dependence of the Doppler effect and the effect of the magnetic-field-induced electron undulations.
- The "natural" bandwidth is the given by the "diffraction grating" effect of the series of magnets in the undulator.
- The angular spread is determined by the fact that the corresponding energy spread cannot exceed the "natural" bandwidth.

Undulator Emission Peak

Consider the undulator magnet array as "seen" by the moving electron in the electron frame (Fig. 2a). The relativistic (Lorentz) frame change rules show one important point: the electron "sees" (Fig. 2b) not only an oscillating magnetic field but also an oscillating electric field in the perpendicular direction — in short, it sees an "electromagnetic wave". Its wavelength in the electron frame, L/γ, equals the undulator period L after Lorentz contraction by the γ-factor.

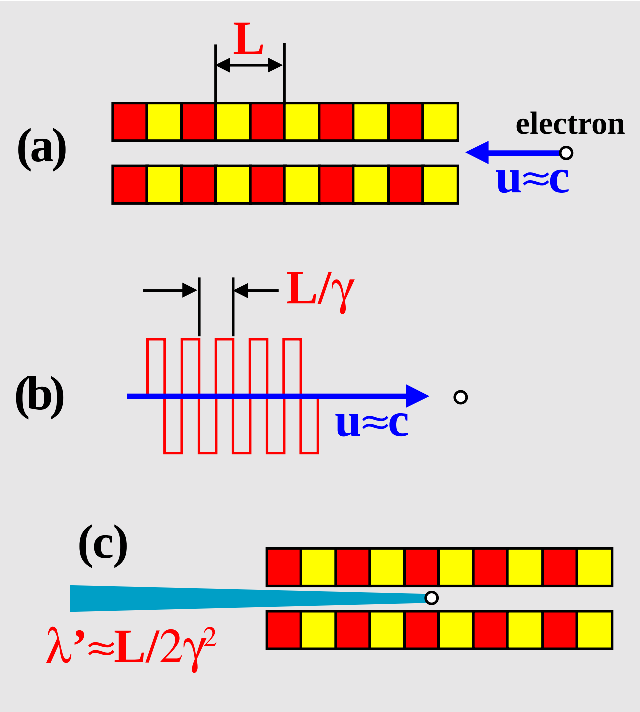


Fig. 2 - Schematic explanation of the on-axis undulator (first harmonic) peak wavelength.

Upon interaction with the "electromagnetic wave", the electron emits (first-harmonic) light of the same wavelength L/γ. Specifically, this is the electron-frame wavelength.

The corresponding wavelength in the laboratory frame is subject to the Doppler effect (Fig. 2c). Along the x-axis ($\theta' = 0$), the multiplying Doppler factor is $\sqrt{1-\beta} \,/\, \sqrt{1+\beta} \approx$ ... [see Eq. A1] ... $\approx 1/2\gamma$. Thus, the emitted first-harmonic wavelength along the x-axis in the laboratory frame is:

$$\lambda' \approx \frac{L}{2\gamma^2} \,. \qquad (1)$$

Thus, the macroscopic undulator period can be transformed into the angstrom-size wavelength of x-rays by a clever use of the two combined relativistic effects; contraction and Doppler shift.

Equations 1 is only a first approximation. With little effort, it can be refined to take into account two important corrections:

- The Doppler effect changes with the emission angle θ'. The correct Doppler multiplication factor is $\gamma(1 - \beta\cos\theta')$. Assuming small angles and $\beta \approx 1$, (see appendix, Eq. A2): $\gamma(1 - \beta\cos\theta') \approx (1/2\gamma)(1 + \gamma^2\theta'^2)$, so that:

$$\lambda' \approx \frac{L}{2\gamma^2}\,(1 + \gamma^2\theta'^2) \quad . \qquad (2).$$

- The "undulations" of the electron with respect to the undulator x-axis decrease its average velocity x-component and therefore the effective β-value and γ-value in the x-direction. The average magnitude of the y-axis component of the velocity is proportional to the acceleration and therefore to the undulator magnetic field magnitude, B. This implies that β^2 effectively decreases becoming $\beta^2(1 - aB^2)$, with a = constant. Likewise, $\gamma^2 = 1/(1 - \beta^2)$ must be replaced by $\gamma^2/(1 + bB^2)$, where b is also a constant.

  Equation 1 becomes then:

  $$\lambda' = \frac{L}{2\gamma^2}(1 + bB^2) \quad , \tag{3}$$

  revealing the important property that the emission peak can be changed by changing B.

The two corrections (Eqs. 2 and 3) can be combined in the first order, obtaining for the undulator peak:

$$\lambda' \approx \frac{L}{2\gamma^2}(1 + \gamma^2\theta'^2 + bB^2) \quad . \tag{4}$$

Undulator Bandwidth

The periodic array of magnets including N. periods acts as a diffraction grating. The relative bandwidth is then given by the well-known diffraction-grating equation. For the first harmonic:

$$\frac{\Delta\lambda'}{\lambda'} = \frac{1}{N} \tag{5}$$

Undulator Angular Spread

Consider, for simplicity, the limit B ≈ 0. The peak λ′ along the x-axis is given by Eq. 1. According to Eq. 2, at an angle δθ′ with respect to the x-axis there is a relative shift:

$$\frac{\Delta\lambda'}{\lambda'} = \frac{\lambda'[1 + \gamma^2(\delta\theta')^2] - \lambda'}{\lambda'} = \gamma^2(\delta\theta')^2 \; ; \qquad (6).$$

on the other hand, $\Delta\lambda'/\lambda'$ cannot exceed the natural limit of Eq. 5, 1/N; thus, the maximum deviation (angular spread) is given by:

$$\delta\theta' \approx \frac{1}{\gamma\sqrt{N}} \quad . \tag{7}$$

We will see later that $(1/\gamma)$ is the "natural" angular spread of all types of synchrotron light emission. In the case of undulators, the actual angular spread is substantially reduced by the factor $\sqrt{N}$.

Bending Magnets and Wigglers

Underlying physics

- The narrow angular spread is the effect of the (relativistic) Doppler modification of the direction of the emitted light.
- The peak emission wavelength is the wavelength corresponding to the cyclotron resonance frequency.
- While estimating the peak wavelength, one must take the relativistic expression for the cyclotron frequency, and the Doppler shift.
- The bandwidth is the given by the "searchlight effect": the narrow angular spread of the emitted light produces short light pulses along the beamline, and therefore broad frequency and wavelength bandwidths.

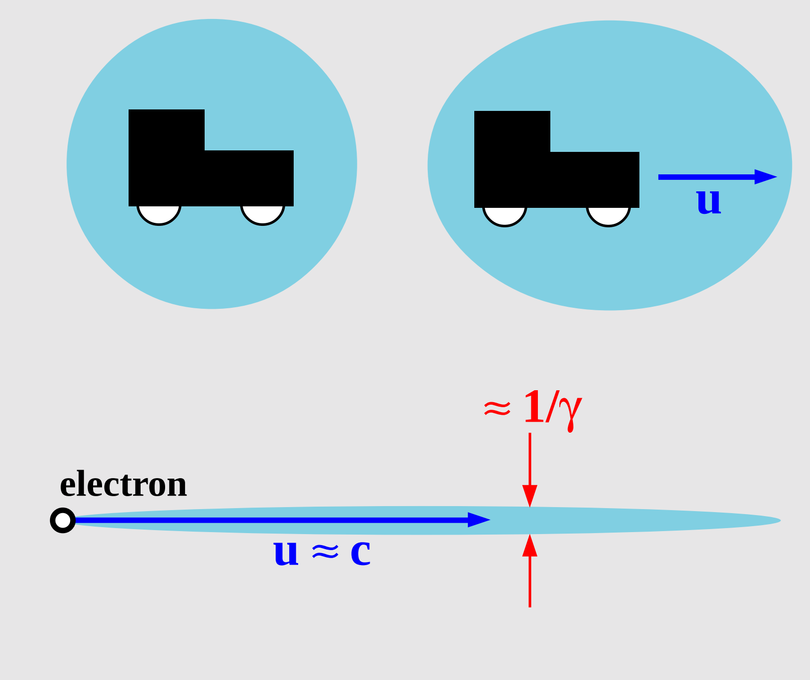
Natural Angular Spread

The extreme collimation (narrow angular spread) of synchrotron light is another aspect of the Doppler effect (Fig. 3). Consider a light beam emitted at the θ-angle in the electron frame. Even for classical waves, e.g., acoustic waves, the change to the laboratory frame would cause a decrease of the angle: $\theta' < \theta$ — and therefore collimation. The reason, for acoustic waves, is trivial: the source speed u is added to the wave speed, "projecting" the wave in the forward direction.

The (relativistic) case of light is slightly more complicated. The light velocity components are dx/dt and dy/dt in the electron frame. The Lorentz frame transformation rules predict a γ-factor for dx' and dt', but not for dy'. Thus, $\theta' \approx (dy'/dt')/(dx'/dt') = dy'/dx'$ is proportional to $1/\gamma$.

This is why the natural angular spread of synchrotron light is related to $1/\gamma$ — and exceedingly small. A little more precisely: $\mathrm{tn}\theta = (dy/dt)/(dx/dt) = dy/dx$; $\mathrm{tn}\theta' = (dy'/dt')/(dx'/dt') = dy'/dx' = dy/[\gamma(dx - c\beta dt)] = \mathrm{tn}\theta / [\gamma(1 - c\beta dt/dx)]$.

Fig. 3 - For acoustic waves, the source motion causes some collimation by “projecting forward” the emission (top). In the case of light and fast-moving electrons, the collimation is much more pronounced and the angular spread $\approx 1/\gamma$ (bottom).



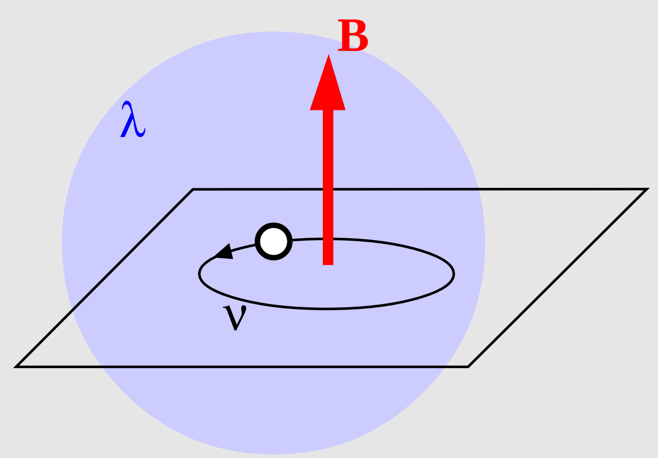
Since $c\,dt/dx = c/(dx/dt) = \cos\theta$, we have:

$$\mathrm{tn}\theta' \approx \frac{\mathrm{tn}\theta}{\gamma(1-\beta\cos\theta)}\ , \qquad (8)$$

so that, for example, an “average” electron-frame emission angle $\theta \approx \pi/4$ becomes an exceedingly narrow $\theta' \approx 0.4/\gamma$.

Bending Magnet Emission Peak

The peak wavelength in the electron frame is related (Fig. 4) to the cyclotron frequency ν: $\lambda = c/\nu$. In turn, the “cyclotron frequency” is the rotation frequency of the magnetic-field-induced motion.



Subject to the Lorentz force caused by a magnetic field, an electron circulates in a circular orbit with a frequency ν, called “cyclotron frequency” and angular velocity $\omega = 2\pi\nu$. Because of the centripetal acceleration, the charged electron emits electromagnetic waves, whose peak frequency is ν, corresponding to a peak wavelength $\lambda = c/\nu$.

In the laboratory frame, the magnetic-field force magnitude is euB, and the cyclotron frequency is:

$$\nu' = \frac{eB}{2\pi m} = \frac{eB}{2\pi\gamma m_o} , \tag{9}$$

where $m = \gamma m_o$ is the relativistic mass.

In the electron frame, the magnetic-field force becomes an electrostatic force of magnitude γeuB. The cyclotron frequency ν must be calculated using this force and the electron rest mass:

$$\nu = \frac{\gamma eB}{2\pi m_o} , \tag{10}$$

thus: $\lambda = \dfrac{2\pi c m_o}{\gamma eB}$.

After Doppler-shifting by the (approximate) factor 2γ, the peak emission in the laboratory frame is:

$$\lambda' \approx \frac{2\pi c m_o}{2\gamma^2 eB} . \tag{11}$$

Note, once again, the characteristic factor $2\gamma^2$.

Bending Magnet Bandwidth

The bending-magnet bandwidth is dominated by the "searchlight" phenomenon. Because of the extreme ($\theta' \approx 1/\gamma$) collimation, each emitting electron behaves like a searchlight. When it passes in front of a beamline connected to a bending magnet source, it illuminates the beamline and the detector only for a very short time. This narrow time pulse corresponds to a broad frequency spectrum, i.e., to a wide wavelength bandwidth.

The wavelength bandwidth $\Delta\lambda'$ corresponding to a pulse duration $\Delta t'$ is given by the Fourier-theorem equation:

$$\Delta t' \Delta\lambda' \approx \lambda'^2 / 2\pi c \tag{12}$$

We must then calculate $\Delta t'$: this is a bit complicated because (Fig. 5) the electron circular motion is combined with the motion of the emitted light. Suppose that at $t' = 0$ the electron position is such that its geometric emission cone $1/\gamma$ begins to touch the detector. If the electron-detector distance is A, the light pulse at the detector begins after the time delay $t_1' = A/c$ required for the light to travel along A.

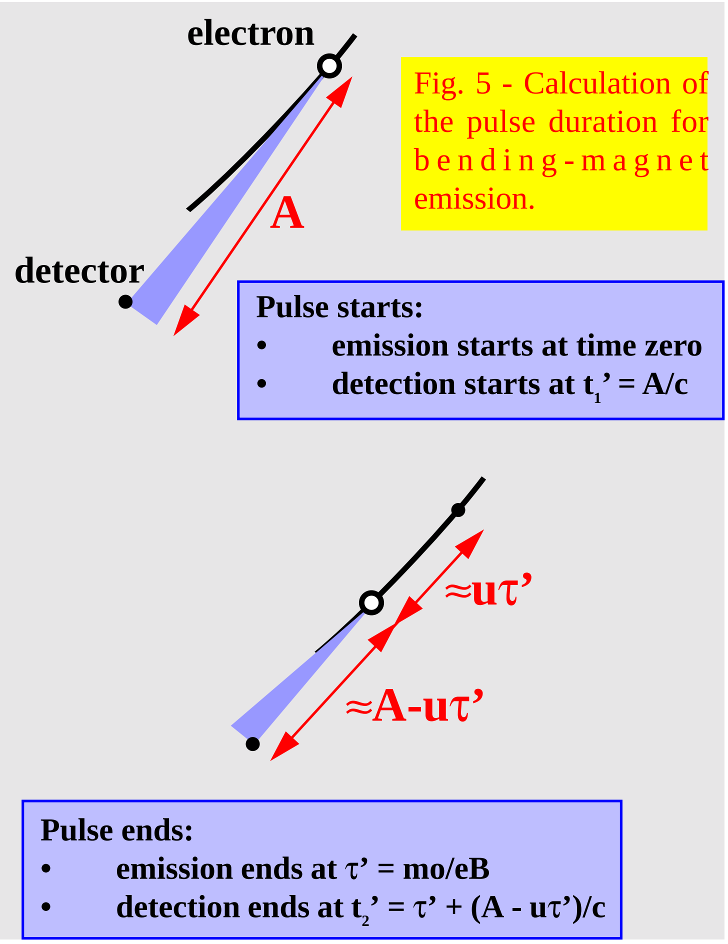


Fig. 5 - Calculation of the pulse duration for bending-magnet emission.

The electron and its geometric emission cone rotate with an angular speed $\omega' = 2\pi\nu'$, where ν' is the cyclotron frequency of Eq. 9. Thus, $\omega' = eB/\gamma m_o$, and the geometric emission cone leaves the detector after a time $\tau' = (1/\gamma)/\omega' = (1/\gamma)/(eB/\gamma m_o) = m_o/eB$.

During the same time, the electron travels reducing its distance from the detector to $\approx A - u\tau'$. Thus, the light pulse at the detector ends at $t_2' = \tau' + (A - u\tau')/c$. And the pulse duration is:

$\Delta t' = t_2' - t_1' = \tau'(1 - u/c) = \tau'(1 - \beta) \approx$ [see Appendix, Eq. A3] $\approx \tau'/2\gamma^2 = m_o/2\gamma^2 eB$. Using Eq. 12:

$$\Delta\lambda' \approx \frac{2\pi c m_o}{2\gamma^2 eB} \quad . \tag{13}$$

Quite interestingly, by comparing Eqs. 11 and 13 we realize that for bending magnets $\Delta\lambda' \approx \lambda'$, and therefore:

$$\frac{\Delta\lambda'}{\lambda'} \approx 1 \ . \tag{14}$$

Critical wavelength

The results of Eqs. 11 and 13 qualitatively correspond to the well-known bending magnet spectra. To recognize this fact, we must remember that the "standard" plot of the bending magnet emission is on a log-log scale.

A broadened peak centered at λ′ and with bandwidth Δλ′ ≈ λ′ does indeed closely resemble the "standard" lineshape of a bending-magnet emission spectrum — see Fig. 6.

Because of Eq. 14, the emission spectrum is characterized by only one parameter, the peak. Quite often, however, a different parameter is used, the "critical wavelength". This is defined as the equipartition wavelength for the emitted energy: equal amounts of emitted energy are located at lower and higher wavelengths.

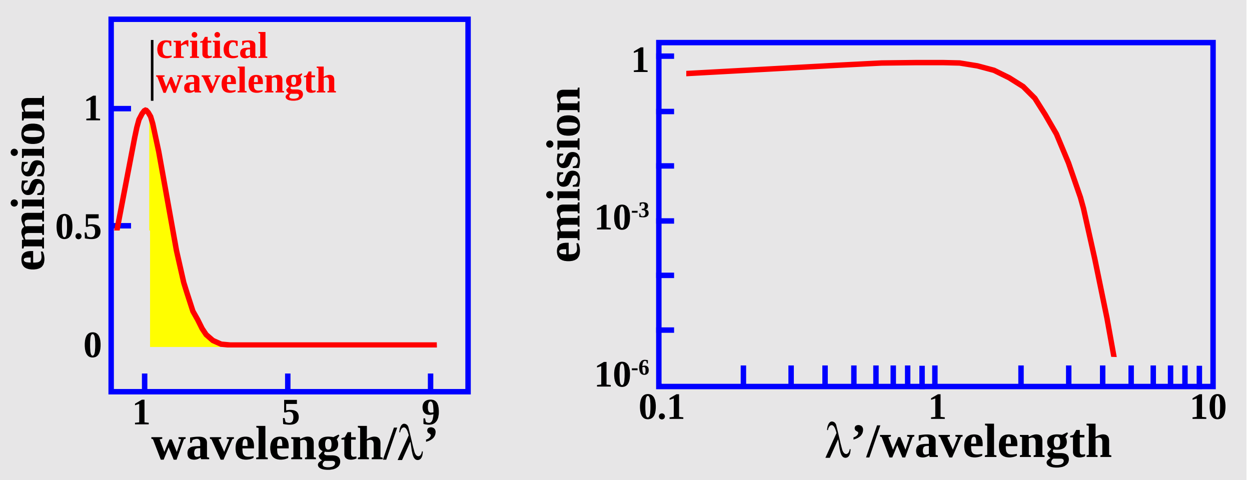


Fig. 6 - Left: plot of the broadened-peak emission of a bending magnet (the horizontal scale is normalized to the peak position). Right: the same plot in a log-log scale, revealing the well-known synchrotron emission spectral lineshape. In the left part, the shaded area emphasizes the fact that the "equipartition wavelength" or critical wavelength is shifted with respect to the peak, because of the limits to calculate the intensity integrals on the left-hand side and on the right-hand side. The same point is emphasized by the vertical line in the left plot, showing the critical wavelength.

When integrating the emitted spectrum to derive the critical wavelength, one should keep in mind (see again Fig. 6) that the integration is, on the left-hand side, from zero wavelength to the peak — whereas on the right-hand side is from the peak to infinite wavelength. This explains why the critical wavelength is shifted to the right-hand side with respect to the peak.

Bending Magnets, Wigglers and Undulators

A comparison of Eqs. 5 and 13 (see Fig. 7) reveals the dramatic difference between undulators and bending magnets: a very narrow bandwidth vs. a very broad bandwidth. What is the cause of this difference?

Simple: we have seen that the broad bending magnet bandwidth is due to the short pulse duration — the beam illuminates the detector only for a short time. Such is not the case of an undulator: the B-field is quite weak, the electron undulations are gentle, and the emission cone illuminates the detector during the entire transit of the electron through the periodic array of magnets.

There exist a third case: if the periodic B-field is strong, the undulations are large and the detector illumination is not a long pulse but a series of short pulses. One finds again the broad bandwidth as for bending magnets. The corresponding emitting device - called "wiggler" - is equivalent to a series of bending magnets.

Flux and Brightness

How can the quality of a light source be assessed? There exist several possible parameters, and this generates some confusion. In order to keep the analysis simple, one must think in terms of the desired final result: roughly speaking, to bring as much light per unit time as possible into the illuminated sample area.

As far as the source is concerned, this corresponds to one or both of two requirements: (1) the total flux of light F emitted by the source must be high; (2) the source must be very "bright". The first requirement is obvious: if more light is emitted, more arrives to the sample. The second requirement is a bit more subtle.

In most cases, the light is brought to the sample by using a series of optical devices along the beamline, such as curved mirrors. Each device can change the beam angular divergence $\Delta\theta'$ and/or the beam size $\Delta y'$. However, optics shows that it cannot change the *product* $\Delta\theta'\Delta y'$.

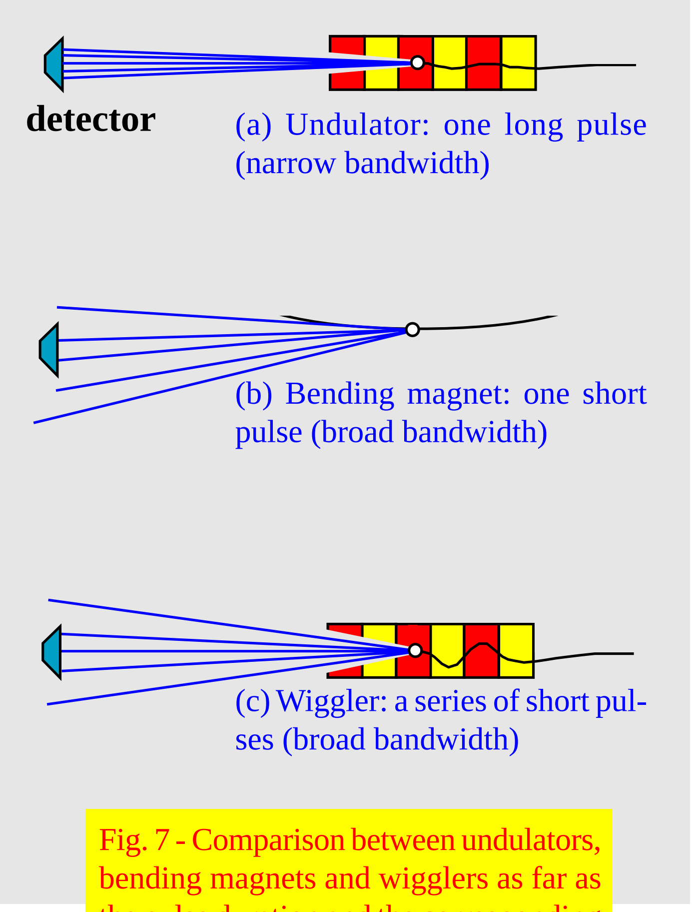


Fig. 7 - Comparison between undulators, bending magnets and wigglers as far as the pulse duration and the corresponding bandwidth are concerned.

The consequences are extremely important: to "focus", i.e., to decrease the beam size on the sample thus increasing the amount of light, one must accept an increase in beam divergence. If the initial (source) value of the $\Delta\theta'\Delta y'$ product is too high, the beam divergence may become so large that, to avoid losing part of the beam, one must increase the size of the optical devices.

However, for x-ray optics large-size devices such as curved mirrors are exceedingly expensive and often unfeasible. It is thus very desirable to use a source with a small value of the $\Delta\theta'\Delta y'$ product.

Note that this conclusion is valid not only for the y-axis, but also for the other direction perpendicular to the x-axis, the z-direction. Calling $\Delta\theta'_y = \Delta\theta'$ and $\Delta\theta'_z$ the angular spreads in the y and z directions, the overall geometric requirement is to have a small value of $(\Delta\theta'_y\Delta y')(\Delta\theta'_z\Delta z')$.

One usually summarizes the requirements of high flux, small size and small angular divergence by saying that one should minimize the value of the parameter:

$$\text{constant} \times \frac{F}{(\Delta\theta'_y\Delta y')(\Delta\theta'_z\Delta z')}\,, \tag{15}$$

which roughly corresponds to the source "brightness".

Thus, a source is "bright" if it has high emitted flux and good geometric characteristics — see Fig. 8.. High brightness is the key parameter defining the astonishing recent progress of synchrotron sources and of their applications.

What causes the high brightness?

The answer is twofold. First, the most recent synchrotron sources have excellent geometric characteristics, The source size ($\Delta y'$ and $\Delta z'$) is primarily determined by the transverse size of the electron beam, which in modern "synchrotrons" (storage rings) is amazingly small. And we have illustrated the effects that make the angular divergence very small for bending magnets (in the horizontal y-direction) and even smaller for undulators.

As to the flux, synchrotrons emits an almost incredible amount of light. Physics teaches that an electron, i.e., an electric charge, which travels along a circular path emits light with a total power proportional to the square of the acceleration. In turn, the total power determines the total flux, so that $F \propto a^2$.

On the other hand, the centripetal acceleration in the electron frame is related to the angular velocity, which in turn is related to the cyclotron frequency: $a = (2\pi\nu)u \approx (2\pi\nu)c$, therefore a^2 is approximately proportional to ν^2. Considering Eq. 10, this means that:

$$F \propto \gamma^2 B^2.$$

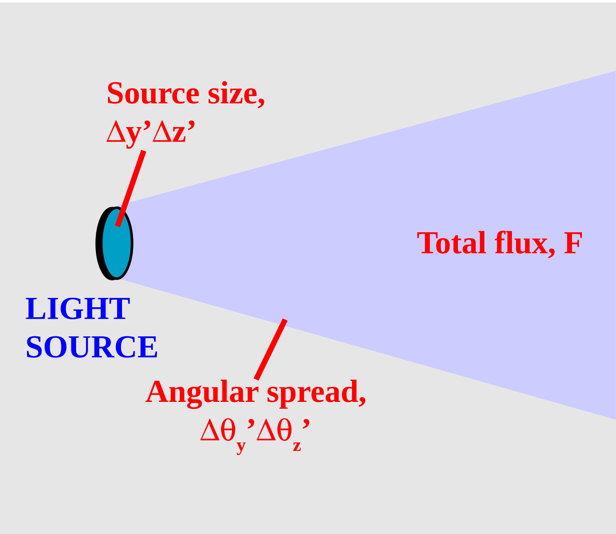


Fig. 8 - The source parameters which determine the brightness or brilliance.

This result is normally expressed using the radius of curvature R of the electron trajectory rather than B. In the laboratory frame, $(2\pi\nu')R = u \approx c$, thus R is proportional to $1/\nu'$ and therefore, according to Eq. 9, to γ/B, so that $B^2 \propto \gamma^2/R^2$, and:

$$F \propto \gamma^4/R^2 .$$

Considering the large magnitude of γ, one can understand the amazing amount of emitted light by a synchrotron — and its rapid increase with the synchrotron (electron) energy, which corresponds to γ.

Polarization

Understanding the polarization of synchrotron light is almost trivial. One should simply imagine the motion of the electrons as seen from the point of view of the detector.

The circular motion along the electron trajectory becomes an oscillatory linear motion when seen from a point of view in the plane of the electron orbit. The electrons "look like" a charge oscillating along an antenna. Thus, in the plane of the ring the emitted light has linear polarization.

If we change slightly this point of view by moving slightly out of the plane of the ring, the electrons appear to move along an elliptical trajectory. The polarization becomes thus elliptical, with different orientation above and below the plane of the ring.

The lateral undulations along a standard undulator will of course produce linear polarization. But undulators can be designed to produce more complicated types of electron motion; for example, "elliptical" undulators give elliptical polarized light.

Coherence

This fundamentally important property of synchrotron light was largely neglected in the past. However, the most recent synchrotron sources possess high coherence, which because of its many applications is extremely important and can no longer be ignored.

Roughly speaking a wave is "coherent" if it can produce *detectable* effects typical of waves — such as diffraction and interference. We can thus analyze coherence by using a specific phenomenon such as diffraction by a pinhole of diameter d (Fig. 9a).

An infinitely small (point) source emitting only one wavelength λ' will of course produce a detectable diffraction pattern. But what happen if the source size is finite, and/or its wavelength bandwidth $\Delta\lambda'$ is finite? The pattern is blurred, and it may or may not still be detectable. The conditions for a detectable pattern define the degree of coherence of the source.

Time (Longitudinal) Coherence

Consider first the effects of a finite wavelength bandwidth — Fig. 9b. For simplicity, we consider a source emitting only two wavelengths, λ' and $\lambda' + \Delta\lambda'$. The angular positions of the first-order diffraction maxima are $\alpha_1(\lambda') \approx 2\lambda'/d$ and $\alpha_1(\lambda' + \Delta\lambda') \approx 2(\lambda' + \Delta\lambda')/d$.

Roughly speaking, the pattern will be still visible, although somewhat blurred, if the shift $\alpha_1(\lambda' + \Delta\lambda') - \alpha_1(\lambda')$ is small with respect to $\alpha_1(\lambda')$, thus:

$$\frac{\alpha_1(\lambda' + \Delta\lambda') - \alpha_1(\lambda')}{\alpha_1(\lambda')} < 1 \, ,$$

which simply gives:

$$\frac{\Delta\lambda'}{\lambda'} < 1 \, . \tag{16}$$

The value of the relative bandwidth, therefore, determines the degree of time coherence. For bending magnet sources (Eq. 14), $\Delta\lambda'/\lambda' \approx 1$, so some kind of additional monochromatization is normally required. On the other hand, even the bandpass action of detectors and/or mirrors may be sufficient to observe the simplest coherence-related effects. For undulators, Eq. 5 reveals an intrinsically high degree of time coherence. However, further monochromatization may be required for the most sophisticated coherence-based techniques.

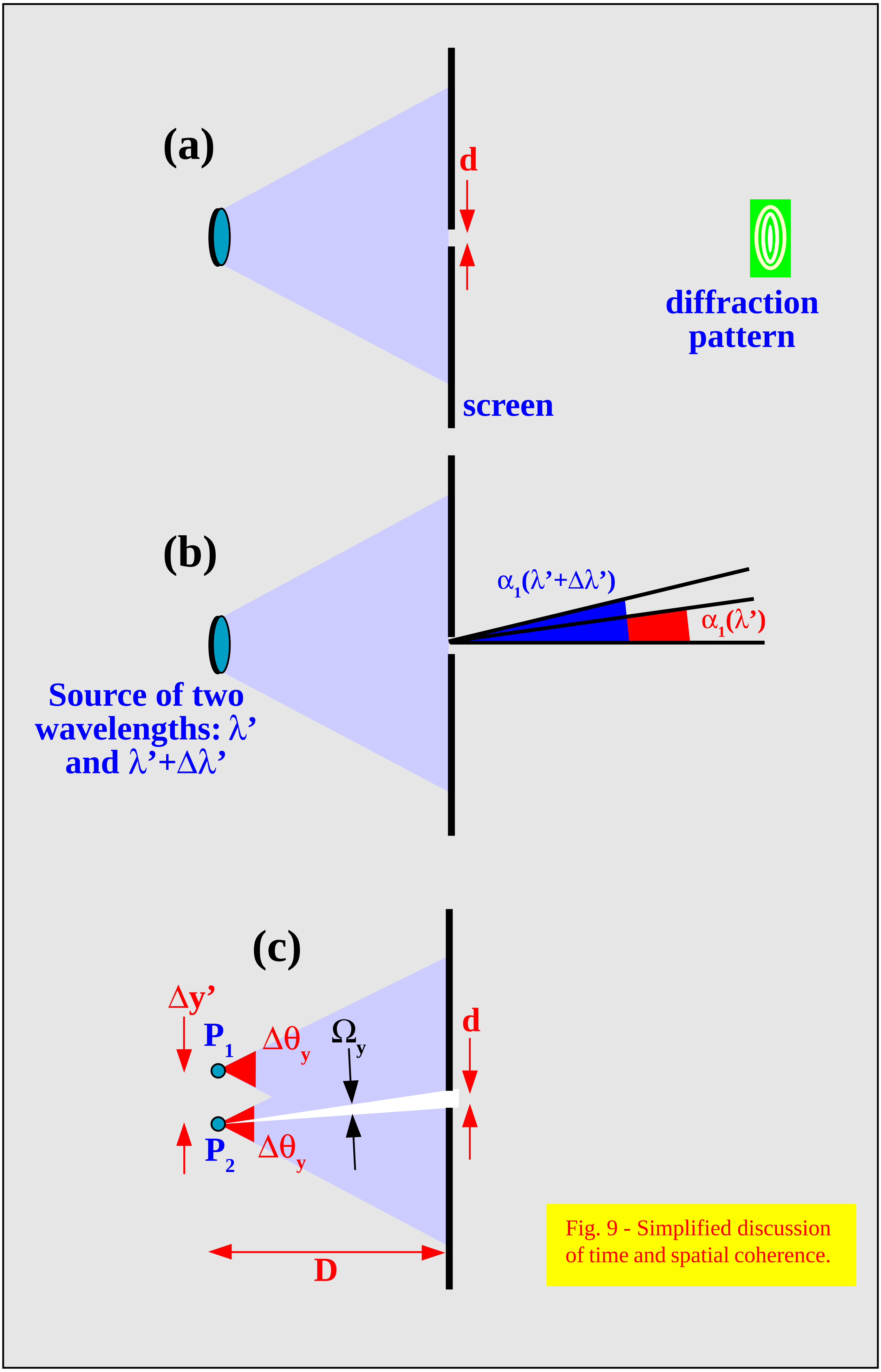


Fig. 9 - Simplified discussion of time and spatial coherence.

Spatial (Lateral) Coherence

We now analyze the effects of the finite size $\Delta y'$ of the source along the y-axis. For simplicity, we consider a source formed by two emitting points P_1 and P_2 (Fig. 9c), at a distance $\Delta y'$ from each other.

The two patterns for P_1 and P_2 are centered at the angles $\alpha_0(P_1) = 0$, and $\alpha_0(P_2) = \Delta y'/D$, where D is the source-pinhole distance. The first-order pattern for P_1 is at $\alpha_0(P_1) = 2\lambda'/d$. Roughly speaking, an overall pattern can still be detected if:

$$\alpha_0(P_2) < \alpha_1(P_1) \ ,$$

which gives:

$$\Delta y' \, (d/D) < 2\lambda' \ .$$

On the other hand, $(d/D) \approx \Omega_y$, the "illumination angle" of the pinhole in the y-direction. Thus, the condition for spatial coherence is:

$$\Delta y' \, \Omega_y < 2\lambda' \ . \tag{17}$$

Therefore, the efforts to improve the brightness by decreasing the source size produce a second, important result: higher spatial coherence.

The decrease in the source angular spread is also a positive factor. Consider again Fig. 9c: each source has angular divergence $\Delta\theta'_y$. Only a fraction of this angle can be used to produce a detectable diffraction pattern. According to Eq. 17, this fraction is $(2\lambda'/\Delta y')/\Delta\theta'_y = 2\lambda'/(\Delta y'\Delta\theta'_y)$. Thus, by decreasing the angular spread one can use more of the source emission to produce coherence-requiring phenomena.

The same analysis is valid for the z-direction. This leads to the definition of the "coherent power" of the source, corresponding to the fraction of the emitted light which can be exploited for coherence-requiring phenomena:

$$\text{Coherent Power} \approx \frac{2\lambda'}{\Delta y'\Delta\theta'_y}\,\frac{2\lambda'}{\Delta z'\Delta\theta'_z} = \frac{2\lambda'^2}{(\Delta y'\Delta\theta'_y)(\Delta z'\Delta\theta'_z)} \ . \tag{18}$$

Note two important points: first, the coherent power decreases with the square of the wavelength, therefore it is difficult to obtain spatially coherent x-rays. Second, Eqs. 15 and 19 show that an improvement of the source geometric factors - size and divergence - increases both the brightness and the spatial coherence.

Diffraction Limit

The efforts to improve the geometric characteristics of a light source, $\Delta\theta'_y$, $\Delta y'$, $\Delta\theta'_z$ and $\Delta z'$, are not open-ended. The ultimate result is the achievement of the so-called "diffraction limit".

Suppose that one must transform a large source of spherical waves into a source with small size and divergence. One simple solution is a screen with a pinhole of size d which eliminates all emission except the light going through the pinhole, so that the beam size is reduced to $\Delta y' \approx \Delta z' \approx d$.

One cannot, however, produce a beam with both infinitely small size and infinitely small divergence. The diffraction by the pinhole spreads the light, giving $\Delta\theta'_y \approx \Delta\theta'_z \approx 2\lambda'/d$. Thus, the minimum possible value for $\Delta y'\Delta\theta'_y$ and $\Delta z'\Delta\theta'_z$ is $\approx 2\lambda'$.

Eq. 18 shows that when $\Delta y'\Delta\theta'_y$ and for $\Delta z'\Delta\theta'_z$ reach this minimum value or "diffraction limit" the coherent power is 100%, i.e., the source has full lateral coherence. The $\Delta y'\Delta\theta'_y$ and $\Delta z'\Delta\theta'_z$ products which determine the brightness cannot be improved beyond the diffraction limit. This is not a technical limitation, but a fundamental optical limit of all light sources.

Where do the actual synchrotron sources stand with respect to this limit? The answer is quite interesting: third-generation sources like Elettra reach the diffraction limit for wavelengths down to $\approx 10^3$ Å. More advanced future facilities like the Swiss Light Source will reach the diffraction down up to $\lambda' \approx 100$ Å, and therefore will constitute up to those wavelengths the ultimate sources as far as geometry is concerned.

Conclusions

All properties of synchrotron sources are the result of relativistic phenomena and classical effects of light emission. Their basic understanding does not require a complicated theoretical treatment, but only the simple use of basic relativistic and optics notions.

Acknowledgments

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Appendix: Useful Mathematical Tricks in Relativity

(1) On-axis Doppler factor:

$$\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}\frac{\sqrt{1+\beta}}{\sqrt{1+\beta}} = \frac{\sqrt{1-\beta^2}}{1+\beta} = \frac{1}{\gamma(1+\beta)} \approx$$

$$\approx .. [\text{for } \beta \approx 1] .. \approx \frac{1}{2\gamma} . \qquad \text{(A1)}$$

(2) Off-axis Doppler factor:

For small angles:

$$\gamma(1 - \beta\cos\theta') \approx \gamma[1 - \beta(1 - (\theta'^2/2))] = \gamma(1 - \beta + \beta\theta'^2/2) = \gamma(1 - \beta) + \beta\gamma\theta'^2/2 =$$

$$= \frac{1-\beta}{\sqrt{1-\beta^2}} + \beta\gamma\theta'^2/2 = \frac{1-\beta}{\sqrt{(1-\beta)(1+\beta)}} + \beta\gamma\theta'^2/2 = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} + \beta\gamma\theta'^2/2 =$$

$$= \frac{\sqrt{1-\beta}\sqrt{1+\beta}}{\sqrt{1+\beta}\sqrt{1+\beta}} + \beta\gamma\theta'^2/2 = \frac{\sqrt{1-\beta^2}}{1+\beta} + \beta\gamma\theta'^2/2 = \frac{1}{\gamma(1+\beta)} + \beta\gamma\theta'^2/2$$

;

For $\beta \approx 1$:

$$\frac{1}{\gamma(1+\beta)} + \beta\gamma\theta'^2/2 \approx (1/2\gamma) + \gamma\theta'^2/2 = (1/2\gamma)(1 + \gamma^2\theta'^2), \text{ thus:}$$

$$\gamma(1 - \beta\cos\theta') \approx (1/2\gamma)(1 + \gamma^2\theta'^2) . \qquad \text{(A2)}$$

(3) Another useful trick:

$$(1 - \beta) = \frac{(1-\beta)(1+\beta)}{1+\beta} = \frac{1-\beta^2}{1+\beta} = \frac{1}{\gamma^2(1+\beta)} \approx ...[\text{for } \beta \approx 1]... \approx \frac{1}{2\gamma^2} . \qquad \text{(A3)}$$